**Unit 1 Cluster 2 (N.RN.3):**

Using properties of rational and irrational numbers (addition, subtraction, and multiplication; **NO DIVISION, NO RATIONALIZING THE DENOMINATOR**)

1.2.1 (N.RN.3) Properties of rational and irrational numbers (i.e. sum of 2 rational numbers is rational, sum of a rational and irrational number is irrational)

**Number Systems**

**Whole Numbers** include zero and the natural numbers

**Rational Numbers** consist of all numbers that can be written as the ratio of two integers

**Imaginary Numbers:** are of the form of *bi* where 

-3*i*, 

**Natural Numbers**

(Counting Numbers)

**Irrational Numbers** consist of all numbers that cannot be written as the ratio of two integers. 

**Integers** are the whole numbers and their opposites (-3, -2, -1, 0, 1, 2, 3, …)

**Real Numbers**

**Complex Numbers:** all numbers of the form *a* + *bi* where *a* and *b* are real numbers. -4 + 3*i*, 2 – *i*

|  |  |  |
| --- | --- | --- |
| **Properties of Real Numbers** | | |
| **Description** | **Numbers** | **Algebra** |
| **Commutative Property**  You can add or multiply real numbers in any order without changing the result. |  |  |
| **Associative Property**  The sum or product of three or more real numbers is the same regardless of the way the numbers are grouped. |  |  |
| **Distributive Property**  When you multiply a sum by a number, the result is the same whether you add and then multiply or whether you multiply each term by the number and then add the products. |  |  |
| **Additive Identity Property**  The sum of a number and 0, the additive identity, is the original number. |  |  |
| **Multiplicative Identity Property**  The product of a number and 1, the multiplicative identity, is the original number. |  |  |
| **Additive Inverse Property**  The sum of a number and its opposite, or additive inverse, is 0. |  |  |
| **Multiplicative Inverse Property**  The product of a non-zero number and its reciprocal, or multiplicative inverse |  |  |
| **Closure Property**  The sum or product of any two real numbers is a real number. |  |  |



**Why can’t I come in?????**

**Sorry we are a CLOSED set!**

|  |  |
| --- | --- |
| **Closure**  When an operation is executed on the members of a set, the result is guaranteed to be in the set. | |
| **Addition:** If two integers are added together, the sum is an integer. Therefore, integers are closed under addition. | **Example:** |
| **Multiplication:** If two integers are multiplied together, the product is an integer. Therefore, integers are closed under multiplication. | **Example:** |
| **Subtraction:** If one integer is subtracted from another, the difference is an integer. Therefore, integers are closed under subtraction. | **Example:** |
| **Division:** If one integer is divided by another integer, the quotient may or may not be an integer. Therefore, integers are not closed under division. | **Example:** |

|  |
| --- |
| **You Decide**  1. What number systems are closed under addition? Justify your conclusions using the method of your choice.  2. What number systems are closed under multiplication? Justify your conclusions using the method of your choice.  3. What number systems are closed under subtraction? Justify your conclusions using the method of your choice.  4. What number systems are closed under division? Justify your conclusions using the method of your choice. |

|  |
| --- |
| **Vocabulary**  For an integer *n* greater than 1, if , then *a* is the ***n*th root of *k***.  Description: http://regentsprep.org/Regents/math/algtrig/ATO3/radpic1.gif  **A radical or the principal *n*th root of *k:*** *k,* the radicand, is a real number. *n,* the index, is a positive integer greater than one. |

|  |
| --- |
| **Properties of Radicals**  **Simplifying Radicals:** Radicals that are simplified have:   * no fractions left under the radical. * no perfect power factors in the radicand, *k*. * no exponents in the radicand, *k*, greater than the index, *n*. |
|  |

|  |  |
| --- | --- |
| Vocabulary  A **prime number** is a whole number greater than 1 that is only divisible by 1 and itself. In other words, a prime number has exactly two factors: 1 and itself. | Example:  *prime*  *not prime* |

Division Rules For a Few Prime Numbers

|  |  |  |
| --- | --- | --- |
| **A number is divisible by:** | **If:** | **Example:** |
| **2** | The last digit is even  (0, 2, 4, 6, 8) | 256 is divisible by 2  255 is not divisible by 2 |
| **3** | The sum of the digits is divisible by 3 | 381 (3+8+1=12 and 12÷3=4) Yes  383 (3+8+3=14 and ) No |
| **5** | The last digit is 0 or 5 | 175 is divisible by 5  809 is not divisible by 5 |
| **7** | If you double the last digit and subtract it from the rest of the number and the answer is:   * 0 * Divisible by 7 | 672 (Double 2 is 4,and ) Yes  905 (Double 5 is 10, and ) No |

Simplifying Radicals

|  |  |
| --- | --- |
| **Method 1:**  **Find Perfect Squares Under the Radical**   1. Rewrite the radicand as factors that are perfect squares and factors that are not perfect squares. 2. Rewrite the radical as two separate radicals . 3. Simplify the perfect square. | **Example:** |
| **Method 2:**  **Use a Factor Tree**   1. Work with only the radicand. 2. Split the radicand into two factors. 3. Split those numbers into two factors until the number is a prime number. 4. Group the prime numbers into pairs. 5. List the number from each pair only once outside of the radicand. 6. Leave any unpaired numbers inside the radical.   Note: If you have more than one pair, multiply the numbers outside of the radical as well as the numbers left inside. | **Example:** |
| **Method 3:**  **Divide by Prime Numbers**   1. Work with only the radicand. 2. Using only prime numbers, divide each radicand until the bottom number is a prime number. 3. Group the prime numbers into pairs. 4. List the number from each pair only once outside of the radicand. 5. Leave any unpaired numbers inside the radical.   Note: If you have more than one pair, multiply the numbers outside of the radical as well as the numbers left inside. | **Example:**   |  |  | | --- | --- | | 5  5 | 75 | | 5 | 15 | |  | 3 | |
|  |  |
|  |  |
|  |  |
| **Method 4:**  **Use Exponent Rules** | **Example:** |
| 1. Rewrite the exponent as a rational exponent. |  |
| 1. Rewrite the radicand as factors that are perfect squares and factors that are not perfect squares. |  |
| 1. Rewrite the perfect square factors with an exponent of 2. |  |
| 1. Split up the factors, giving each the rational exponent. |  |
| 1. Simplify. |  |
| 1. Rewrite as a radical |  |
| **Method 4 with Variables:** | **Example:** |
| 1. Rewrite the exponent as a rational exponent. |  |
| 1. Rewrite the radicand as two factors. One with the highest exponent that is divisible by the root and the other factor with an exponent of what is left over. |  |
| 1. Split up the factors, giving each the rational exponent. |  |
| 1. Rewrite the exponents using exponent rules. |  |
| 1. Simplify. |  |
| 1. Rewrite as a radical |  |

|  |  |
| --- | --- |
| **Adding and Subtracting Radicals**  *To add or subtract radicals, simplify first if possible, and then add or subtract “like” radicals.* | |
| 1. They both have the same term under the radical so they are “like” terms. | **Example:** |
| 2. Add the coefficients of the radicals. |  |
|  |  |
| 1. They both have the same term under the radical so they are “like” terms. | **Example:** |
| 2. Subtract the coefficients of the radicals. |  |
|  |  |
| 1. They are not “like” terms, but one of them can be simplified. | **Example:** |
| 2. Rewrite the number under the radical. |  |
| 3. Use the properties of radicals to write the factors as two radicals. |  |
| 4. 25 is a perfect square and the square root of it is 5. |  |
| 5. Multiply the coefficients of the second radical. |  |
| 6. Now they are “like” terms, add the coefficients. |  |
|  |  |
| 1. None of them are “like” terms. Simplify if you can. | **Example:** |
| 2. Factor each number inside the radical. |  |
| 3. Use the properties of radicals to simplify. |  |
| 4. 4 and 9 are perfect squares; their square roots are 2 and 3. |  |
| 5. Multiply the numbers outside of the radical. |  |
| 6. Only the terms with are “like” terms. |  |
| 7. Simplify. |  |
| 1. They are not like terms, but they can be simplified. | **Example:** |
| 2. Rewrite the expressions under the radical. |  |
| 3. Use properties of radicals to rewrite the expressions. |  |
| 4. The cube root of 8 is 2 and the cube root of 27 is 3. |  |
| 5. Multiply the coefficients of the last radical. |  |
| 6. Add or subtract the coefficients of the like terms. |  |
| 7. Simplify. |  |
|  |  |
|  |  |
|  |  |
|  |  |
| 1. They are not like terms, but one of them can be simplified. | **Example:** |
| 2. Rewrite the expression under the radical. |  |
| 3. Use properties of radicals to rewrite the expression. |  |
| 4. 8 and are perfect cubes. The cube root of 8 is 2 and  is *x*. |  |
| 5. Multiply the coefficients of the first radical. |  |
| 6. Now they are like terms, add the coefficients of each. |  |
| 7. Simplify. |  |

|  |  |
| --- | --- |
| Multiplying Radicals  **Multiplying radicals with the same index** | |
|  | **Example:** |
| 1.Multiply the coefficients and multiply the numbers under the radicand. |  |
| 2. If possible, simplify. This is already simplified. |  |
|  | **Example:** |
| 1. Use the distributive method to multiply. |  |
| 2. Use properties of radicals to simplify. |  |
| 3. Simplify any radicals. |  |
| 4. Combined “like” terms if possible. |  |
|  | **Example:** |
| 1. Use the distributive method to multiply. |  |
| 2. Use properties of radicals to simplify. |  |
| 3. Simplify and combine “like” terms. |  |
| 4. The square root of 25 is 5. |  |
| 5. Combine like terms. |  |
|  |  |
|  | **Example:** |
| 1. Use the distributive method to multiply. |  |
| 2. Use properties of radicals to simplify. |  |
| 3. Simplify and combine like terms. |  |
| 4. The square root of 4 is 2. |  |
| 5. Simplify. |  |
|  | 41 |

|  |  |
| --- | --- |
| **Multiplying Radicals with Different Indices**  *Note: In order to multiply radicals with different indices the radicands must be the same.* | |
|  | **Example:** |
| 1. Rewrite each radical using rational exponents. |  |
| 2. Use properties of exponents to simplify. |  |
| 3. Combine the fractions by finding a common denominator. |  |
|  | **Example:** |
| 1. Rewrite each radical using rational exponents. |  |
| 2. Use properties of exponents to simplify. |  |
| 3. Combine the fractions by finding a common denominator. |  |
|  | **Example:** |
| 1. Rewrite the inner radical using rational exponents. |  |
| 2. Rewrite the outer radical using rational exponents. |  |
| 3. Use properties of exponents to simplify. |  |
| 4. Simplify by multiplying fractions. |  |

**Practice Exercises A**

Add or Subtract

1. 

2. 

3. 

4. 

5. 

6. 

7. 

8. 

9. 

**Practice Exercises B**

Multiply and simplify the result.

|  |  |  |
| --- | --- | --- |
| 1. | 5. | 9. |
| 2. | 6. | 10. |
| 3. | 7. | 11. |
| 4. | 8. | 12. |

**You Decide:**

1. Add: . Can you write the result as the ratio of two numbers? (Use your graphing calculator to change the sum from a decimal to a fraction by pushing the math button and select FRAC)

2. Add: . Can you write the result as the ratio of two numbers?

3. Add: . Can you write the result as the ratio of two numbers?

4. Add: . Can you write the result as the ratio of two numbers?

5. Add: . Can you write the result as the ratio of two numbers?

6. Add: . Can you write the result as the ratio of two numbers?

7. Write a rule based on your observations with adding rational and irrational numbers.